# Tandem Queue Decomposition: A Throughput-Optimal Routing Policy for **Quantum Key Distribution Networks**



- Quantum key distribution (QKD) enables two geographically separate communicating parties to exchange symmetric private keys, whose information-theoretical security is guaranteed by the fundamental principles of quantum mechanics
- Top figure shows Alice and Bob equipped with a quantum link and classical link exchanging a sufficiently long quantum key k through quantum link to encrypt the message *m* that is sent over the classical link
- We consider trusted node architecture where nodes are assumed to be secure and only the links can be compromised. The architecture is shown in the figure below



#### Model

- We consider a network with arbitrary topology, represented by a graph  $\mathscr{G}(V, E)$ , where V denotes the set of nodes (|V| = n) and E denotes the set of edges (|E| = m)
- Physical link capacity is  $\gamma_e$  and quantum key generation rate at  $\eta_e$ . for edge *e*. Packet arrival is assumed to be i.i.d. across the time slots

## Tandem Queue Decomposition



2: (Route Assignment) For all incoming packets, compute a Min-Weight Route in the weighted graph  $\mathscr{G}(V, E)$ .

3: (Key Generation) Generate symmetric private keys for every edge  $e \in E$  via QKD and store them in key banks.

4: (Encryption) Encrypt the data packets waiting in the physical queue  $X_e$  with available keys in the key bank and internally transfer the encrypted packets to the downstream queue  $Y_{\rho}$  for every edge  $e \in E$ .

5: (Packet Forwarding) Forward the encrypted packets from queue  $Y_e$  to the queue  $X_{e'}$  for every edge *e* according to some packet scheduling policy (ENTO [3], FIFO, etc.). Here e' is the next edge in the assigned route of a packet.

6: (Decryption) Decrypt the data packets received at physical queue  $X_e$  for every edge *e* using the symmetric key generated earlier via the QKD process.

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### Algorithm

Tandem Queue Decomposition (TQD) Policy at slot *t* 

**Require:** Graph  $\mathscr{G}(V, E)$ , Virtual Queue lengths  $\{\tilde{X}_{e}(t), \tilde{Y}_{e}(t), \forall e \in E\}$  at the slot t

1: (Edge-Weight Assignment) Assign each edge  $e \in E$  a weight  $W_e(t) \leftarrow X_e(t) + Y_e(t)$ .

7: (Updating the Virtual Queues) Update the virtual queues assuming a precedence- relaxed system, i.e.,

> $\tilde{X}_e(t+1) \leftarrow \left(\tilde{X}_e(t) + A_e^{\pi}(t) - \kappa_e(t)\right)^+, \forall e \in E$  $\tilde{Y}_e(t+1) \leftarrow \left(\tilde{Y}_e(t) + A_e^{\pi}(t) - \gamma_e(t)\right)^+, \forall e \in E.$



	1000	
n Delay (Time-slot)	800	
	600	
	400	
Mea	200	
	0	1

Backpressure performs poorly due to small congestion gradients and poor in-network residual key management at lower & higher arrival rates respectively

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• Left figure shows the variation of mean packet delay as a function of the mean arrival rate  $\lambda$  for unicast flow

### Extension to Multilevel Security



• Denote  $S^*$  to be the group of users requiring quantum encryption. In practice, some packets  $S \setminus S^*$  might not require quantum encryption. Those skip the  $X_e$  and join the  $Y_e$  queues directly

•  $E_S \subseteq E$  be the set of edges equipped with the QKD module. The shortest path is computed on the induced graph  $\mathscr{G}(V, E_S)$  for packets originating from some source in the set  $S^*$ 

• Packets from other sources are routed along the edges that lack QKD module. We call this extension to be *e*-TQD

#### References

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