Fundamental Limits of Age-of-Information in Stationary and Non-stationary Environments

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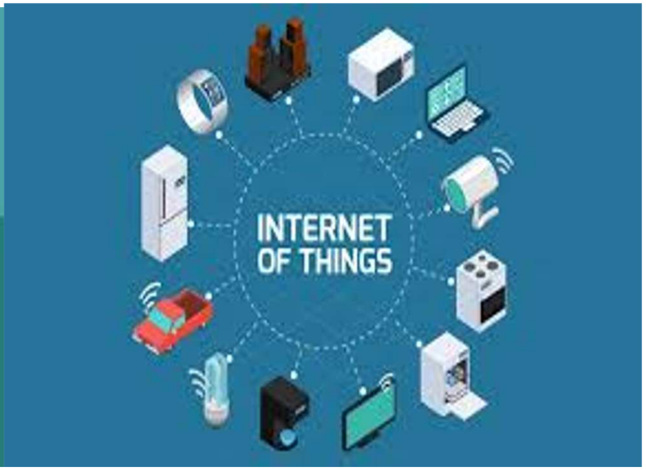
Age of Information: a metric for QoE

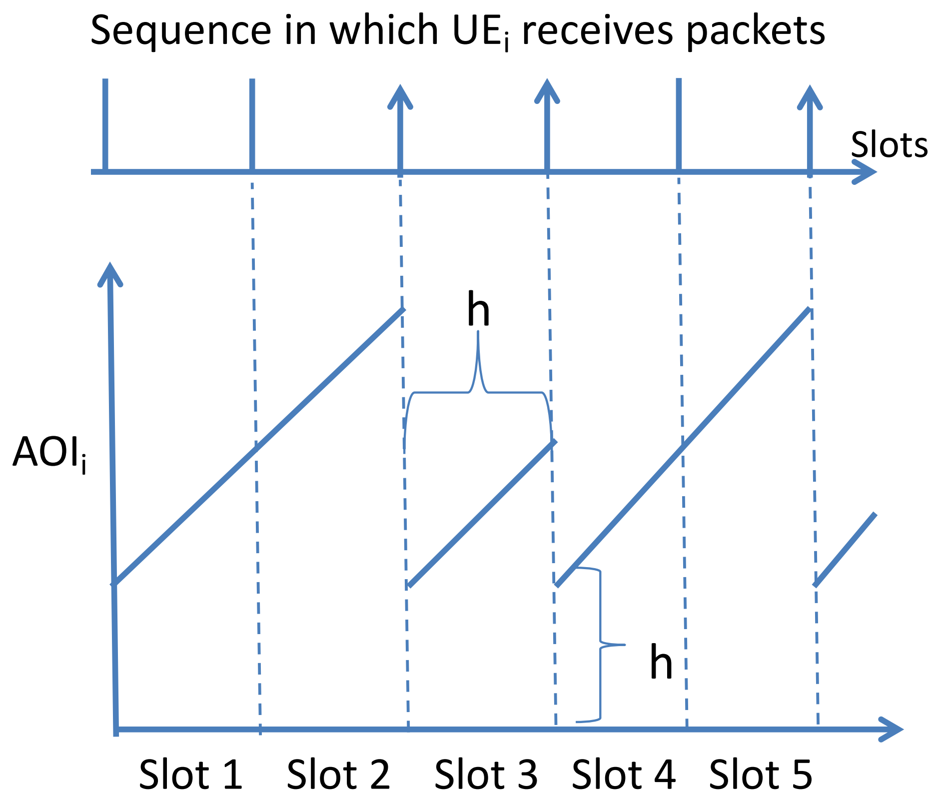
- The **Quality of Experience (QoE)** for the users plays a major role in today's network design:
  - real-time AR and VR systems
  - Internet of Things (IoT)

- **Age-of-Information (AoI)** is a newly proposed metric to measure the ***freshness*** of information available to the end-users

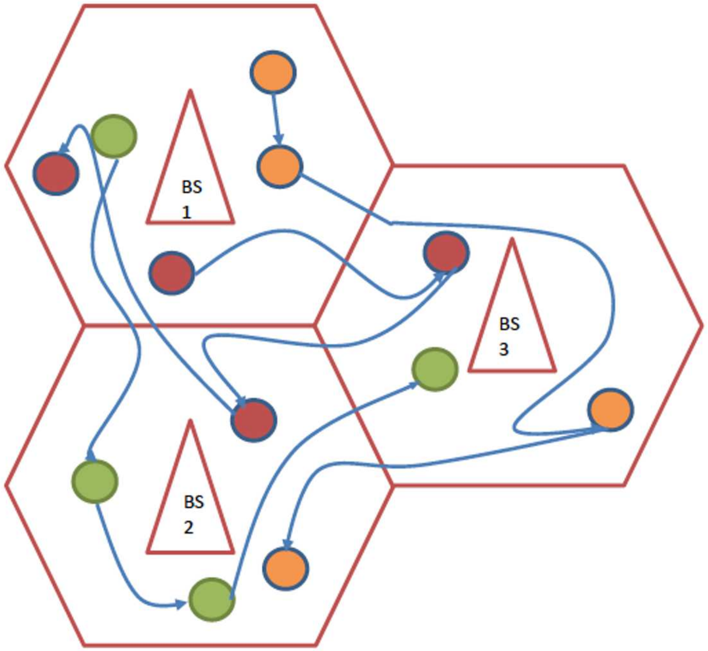


Age-of-Information

- **AoI of the i^{th} user at time t: last time before t at which the i^{th} user received a packet successfully from the Base Station.**
- $h_i(t) = t - t_i(t)$



Scenario I – Stationary environment



- *N* UE's move around independently in an area with *M* Base Stations.
- Time is slotted
- Channel from UE_i to any BS is a Binary Erasure Channel (BEC) with success probability p_i
- At every slot, each BS beamforms and schedules a packet transmission to one of the UEs in its coverage area

Performance Metric

- **Minimize Long-term average AoI of users**

$$\text{AoI}^* = \inf_{\pi \in \Pi} \limsup_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \frac{1}{N} \left(\sum_{i=1}^{N} \mathbb{E}^{\pi}(h_i(t)) \right)$$

  - **An average-cost MDP with a countably infinite state space**
  - **Design efficient policies with a guaranteed approximation ratio**

Lower Bound on the Optimal AoI (Converse)

- **Theorem 1: Under any scheduling policy, AoI is lower bounded as:**

$$\text{AoI}^* \geq \frac{1}{2Ng(\psi)}\left(\sum_{i=1}^{N}\sqrt{\frac{1}{p_i}}\right)^2 + \frac{1}{2}$$

- $g(\psi)$ **is the expected number of cells with at least one UE**
- $g(\psi)$ **captures the *multi-user diversity* of the system**
  - **Under uniform mobility, we have a closed form expression:**

$$g(\psi) = \sum_{j=1}^{M}\left(1 - \prod_{i=1}^{N}(1-\psi_{ij})\right)$$

- **Since** $g(\psi) \leq \min\{M, N\}$ **we have:**

$$\text{AoI}^* \geq \frac{1}{2N\min\{M,N\}}\left(\sum_{i=1}^{N}\sqrt{\frac{1}{p_i}}\right)^2 + \frac{1}{2}$$

Achievability

- π^{MMW} (Multi-cell Max-Weight policy): At every slot, each BS schedules a UE under its coverage that has the highest index among all other UEs, where the index is defined as

$$I_i(t) \equiv p_i h_i^2(t)$$

- Theorem 2: π^{MMW} is a 2-approximation scheduling policy for statistically identical UEs with i.i.d. uniform mobility

- The proof proceeds by constructing a linear Lyapunov function of the AoI's:

$$L(\mathbf{h}(t)) = \sum_{i=1}^{N} \frac{h_i(t)}{\sqrt{p_i}}$$

- Then upper bound the value of the conditional drift for MMW policy:

$$\mathbb{E}\big(L(\mathbf{h}(t+1)) - L(\mathbf{h}(t))|\mathbf{h}(t), \mu(t), \mathbf{C}(t)\big)$$

- Finally, we use the converse result to show $\text{AoI}^{\text{MMW}} \leq 2\text{AoI}^*$

Scenario II –Non-stationary environment

- We consider an adversarial framework for modeling the non-stationary environment

- *N* UEs are under the coverage of a single BS
  – For UEs with mobility, please refer to our recent paper "Competitive Algorithms for minimizing the Maximum Age-of-Information", R. Bhattacharjee, A. Sinha, MAMA 2020, Boston, MA, (https://arxiv.org/abs/2005.05873).

- The channel state of any UE_i at any time slot t could be in either Good or Bad states.
  – If the BS schedules a packet to a channel in a good state at that slot, packet in transmitted successfully

- The channel states are chosen by the omniscient adversary at every time slot *t*

- Online scheduling policy has no information on the channel states for the current or future slots

Performance Metric

- **Cost function for a *T*-length horizon :** $\text{AoI}\ (T) = N^{-1} \sum_{t=1}^{T} \left(\sum_{i=1}^{N} h_i(t) \right)$
- $\sigma \in \{\{0,1\}^N\}^T$ **is a sequence of length *T* representing the vector of channel states chosen by the adversary**
- **Competitive ratio of any online policy** $\mathcal{A}$ **is defined as:**

$$\eta^{\mathcal{A}} = \sup_{\sigma} \left(\frac{\text{Cost of the online policy } \mathcal{A} \text{ on } \sigma}{\text{Cost of OPT on } \sigma} \right)$$

- **The offline optimal policy OPT is assumed to be equipped with full knowledge on the entire channel-state sequence**
- **Our objective is to design a policy with a <u>small competitive ratio</u>**

Achievability and Converse

- **Max-Age policy (MA): MA schedules the user having the highest age**
  - **i.e., UE scheduled at time t:** $\arg\max_i h_i(t)$

- **Theorem (Achievability): In the adversarial setting with N users, the MA policy is** $\eta = O(N^2)$ **competitive for minimizing the average AoI.**
  - **The first policy with a provably finite competitive ratio for AoI**

- **Using Yao's minimax principle, we also have the following lower bound**

- **Theorem (Converse): For any online policy, the competitive ratio is lower bounded as**

$$\eta \geq \frac{N}{2} + \frac{1}{2N}$$

- **Hence, MA is competitively optimal up to a factor of** $O(N)$ **for average age. In a recent paper, we show that MA is competitively optimal up to a factor of**
- $O(\log N)$ **for the max-age metric.**

Proof outline for Achievability

- Let the MA policy have a total of K successful transmissions

- Divide the divide the time horizon into K successive intervals, with the i^{th} interval denoting the time between i-1^{th} and i^{th} successful transmissions:

$$\Delta_i \equiv T_i - T_{i-1}$$

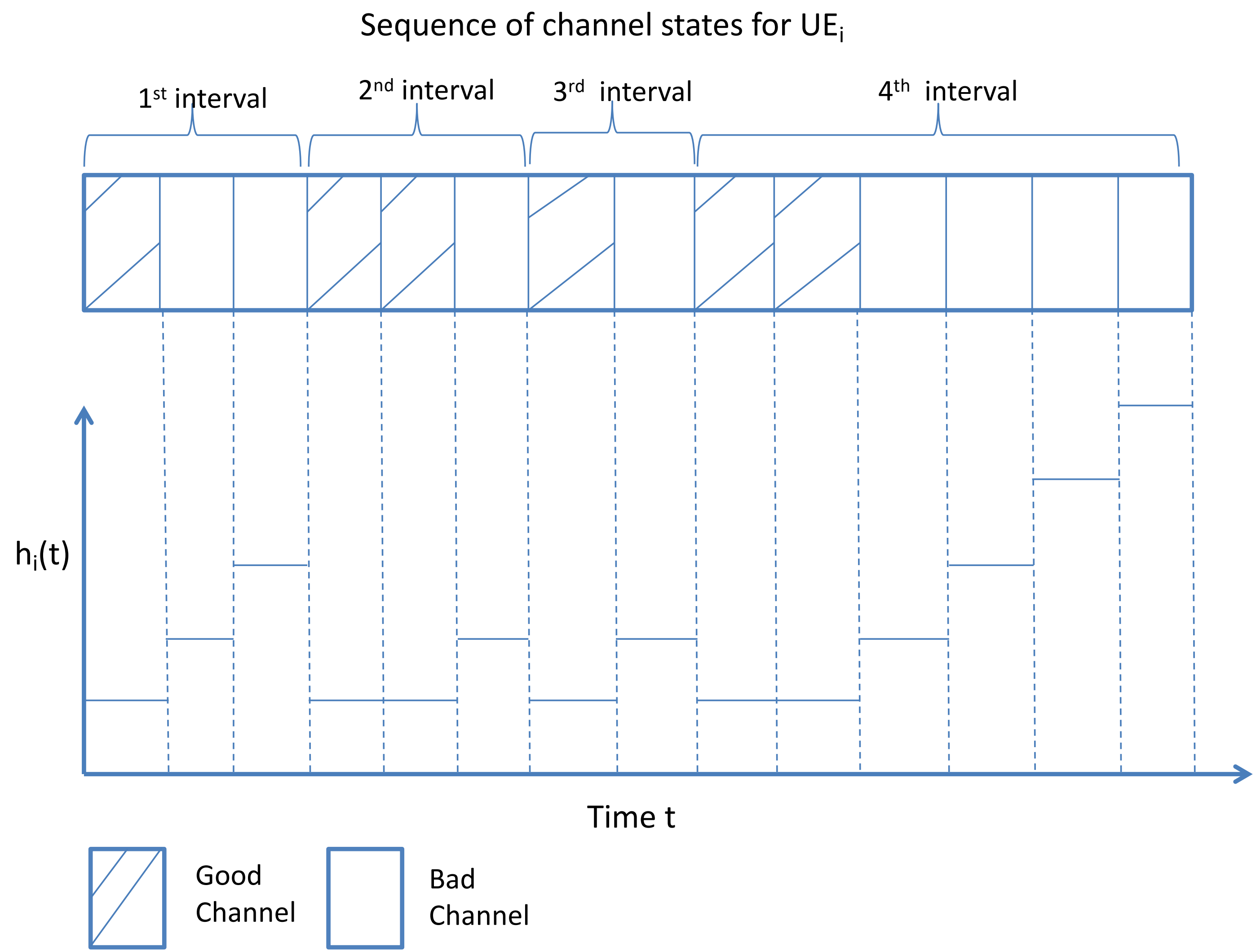
- Observation 1: At a slot, if MA policy transmits successfully ⇒ the optimal policy **OPT** also transmits successfully

- Observation 2: MA policy is a persistent round robin policy i.e. it keeps on scheduling the Max-age user until its transmission is successful

- At the end of k^{th} slot of the i^{th} interval, the (sorted) ages of UE's under MA is given by :

$$\{k, k+\Delta_{i-1}, k+\Delta_{i-1}+\Delta_{i-2}, \ldots, k+\sum_{j=1}^{N-1} \Delta_{i-j},\ 1 \leq k \leq \Delta_i.\}$$

Illustration of the "Intervals"



An illustration of the intervals. We upper bound the cost incurred by MA and lower bound the cost incurred by OPT in the intervals. Details in the paper.

Proof outline of Achievability

- **Lower bound of the cost incurred by OPT in the i**th **interval:**

$$C_i^{\mathsf{OPT}} \geq (N-1)\sum_{k=1}^{\Delta_i} 1 + \sum_{k=1}^{\Delta_i}(1+k)$$

- **Upper bound of the cost incurred by MA in the i**th **interval:**

$$C_i^{\mathsf{MA}} = \sum_{k=1}^{\Delta_i} k + \sum_{k=1}^{\Delta_i}\sum_{m=1}^{N-1}\Big(k + (\sum_{j=1}^{m}\Delta_{i-j})\Big) \leq \frac{N}{2}\Big(N\Delta_i^2 + \Delta_i + \sum_{j=1}^{N-1}\Delta_{i-j}^2\Big)$$

- **Finally, sum over the costs over all intervals to upper bound the competitive ratio:**

$$\eta^{\mathsf{MA}} = \frac{\sum_{i=1}^{K} C_i^{\mathsf{MA}}}{\sum_{i=1}^{K} C_i^{\mathsf{OPT}}} \leq 2N^2$$

Proof outline for the Converse

- Consider the following distribution p of the channel states for applying Yao's principle:

  - At every slot t, a UE is chosen independently and uniformly at random, and assigned a Good channel.
  - The rest of the UEs are assigned Bad channels

- Under p, only one channel is in Good state at every slot.

- It can be seen that the sequence of age variables constitute a Renewal Process for each user.

- A generic renewal interval for any user consists of two parts:

  - A consecutive sequence of Good channels of length τ_G (Geometric R.V.)
  - A consecutive sequence of Bad channels of length τ_B (Geometric R.V.)

$$\begin{aligned} \mathbb{P}(\tau_{\mathsf{G}} = k) &= q^{k-1}(1-q), \quad k \geq 1 \\ \mathbb{P}(\tau_{\mathsf{B}} = k) &= q(1-q)^{k-1}, \quad k \geq 1 \end{aligned}$$

Proof of Converse contd.

- The cost incurred by OPT for the i^th user in any renewal interval is given by:

$$c_i(\tau) \;=\; c_i(\tau_{\mathsf{G}}) + c_i(\tau_{\mathsf{B}}) \;=\; \sum_{t=1}^{\tau_{\mathsf{G}}} 1 + \sum_{t=1}^{\tau_{\mathsf{B}}} (1+t) \;=\; \tau_{\mathsf{G}} + \frac{3}{2}\tau_{\mathsf{B}} + \frac{1}{2}\tau_{\mathsf{B}}^2.$$

- Using the Renewal Reward Theorem, the time-averaged expected cost of the i^th user is:

$$\lim_{T\to\infty} \frac{\mathbb{E}(C_i(T))}{T} = \frac{\mathbb{E}(c_i(\tau))}{\mathbb{E}(\tau)} = \frac{1}{q} = N, \quad \forall i$$

- Thus, the time-averaged expected cost incurred by OPT $\bar{C}(\mathsf{OPT}) = N^2$

- Lower bound of the time-averaged expected cost for any online scheduling policy: use previous (stochastic) converse result by taking

$$p_i = 1/N, \forall i$$

- Dividing the lower bound of cost of any online policy by upper bound of OPT's cost and using Yao's minimax principle, we have the lower bound

AoI minimization with Channel Predictions

- Under the adversarial channel model, any online scheduling policy has a worst-case competitive ratio growing at least linearly with the number of users

- We now exploit the fact that wireless channels with block-fading may often be estimated quite accurately for a few subsequent future slots

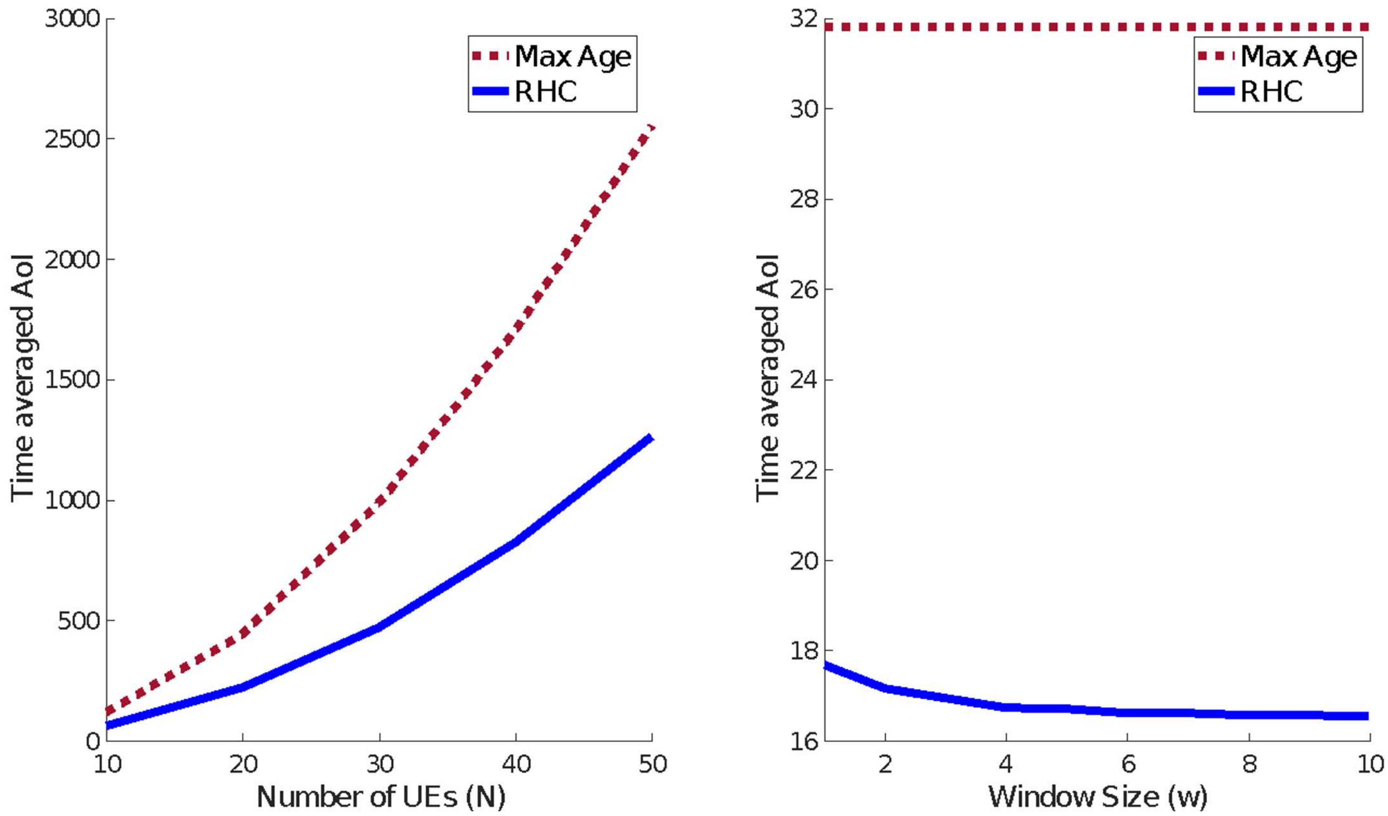
- Relaxed adversarial model: At any slot t, the BS can estimate the channels perfectly for a window of the next $w \geq 0$ slots.

- w is a parameter that depends on coherence time of the channel

  – $w=0$ corresponds to the adversarial model discussed previously

- Policy: Receding Horizon Control (RHC) – solves a Dynamic Program with the horizon set to the next w slots.

Numerical Simulations



- We conclude that even a limited channel estimation capability **has a great RoI for minimizing the AoI**!

Summary

- **Stationary Channel Model with mobile users: We obtain a 2-optimal scheduling policy for statistically identical UEs with i.i.d. uniform mobility**

- **Non-stationary Channel: We have proposed a new adversarial model for the non-stationary environments**

- **Achievability: We show Max Age policy is $O(N^2)$-competitive in the adversarial setting**

- **Converse: We show that the lower bound on competitive ratio in the adversarial grows linearly in the number of UEs**
  - **Open Problem: Tighten the gap between the upper and lower bounds**

- **Extension: We have proposed an extension of the adversarial model, taking into account the future channel state estimations**
  - **Open Questions: Variation of the competitive ratio with *w* in the relaxed model**

For any questions/comments, please reach out to me at

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Thank you!